Dynamical Mass generation in QED3

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Chapter 6

Summary and Conclusions

Quantum field theories in three space-time dimensions are interesting toy models for the investigation of phenomena that play a role in the physical, four-dimensional world. One of the fascinating puzzles encountered in elementary particle physics concerns the origin of mass. In four-dimensional models like QED_4 and QCD_4, masses can be generated in a way which has its origin in the dynamics of the model. However, ultraviolet divergences make the analysis of the infrared region difficult, and this is the important region for dynamical mass generation. Three-dimensional models have less severe ultraviolet problems and therefore offer the possibility to concentrate on infrared effects. The hope is that the methods developed to cure the infrared problems in three dimensions will finally contribute to a better understanding of dynamical mass generation in four-dimensional models.

In this thesis we have investigated dynamical fermion mass generation in QED in three dimensions, with N two-component fermion flavours. We have made use of Dyson-Schwinger equations, the non-perturbative equations of motion, since the dynamical generation of mass is an essentially non-perturbative phenomenon.

We have started, in Section 4.1, with the analysis of the case $N = 1$. Using the unusual (in this field of research), but surprisingly effective method of Fourier transformation, we have shown that dynamical fermion mass generation occurs for any value (including zero) of the bare photon mass μ. Our results provide a counter-example to the generally held view in the literature, that dynamical mass generation is not possible in the case of an odd number N of two-component fermions. A more careful analysis is needed for the general odd-N case, with N preferably *not* in the large-N limit, to find a definitive answer as to whether or not dynamical mass generation occurs.

In Sections 4.3 to 4.6, we have investigated the model for general N, with N large, i.e., we have employed an expansion to leading (and next-to-leading) order in $1/N$. In the limit $N \to \infty$ (with e^2N constant), the model QED$_3$ simplifies enormously, in that the full photon propagator, appearing in the self-energy term in the fermion Dyson-Schwinger equation, equals its bare equivalent, multiplied only by a simple factor due to a one-loop fermion correction. This effective form for the photon propagator in the large-N limit softens the infrared behaviour, since the one-loop vacuum polarization acts as an infrared cut-off.

In Chapter 4 we have considered several Ansätze that have been used in the literature for the fermion-photon vertex in the fermion Dyson-Schwinger equation. The results are summarized here.

Appelquist et al. [48-50] assume a bare vertex and neglect wave-function renormalization. They find a critical value for the coupling λ ($\sim 1/N$) at $\lambda_c = \frac{1}{4}$, which corresponds in a two-dimensional representation to a critical value for N at $N_c \approx 6.48$. For $\lambda > \lambda_c$ they find a non-zero dynamical fermion mass, whereas for $\lambda < \lambda_c$, only the trivial solution ($m = 0$) is allowed. Their results are claimed to be confirmed numerically by Dagotto, Kogut and Kocic [52]. However, these numerical results are not convincing.

Pennington et al. [53,54] criticize the approach of Appelquist et al. on the point of the neglect of wave-function renormalization. Multiplying the bare vertex γ_ν with the wave-function renormalization function $\beta(p)$, as motivated by the Ward-identity, and treating the coupled set of equations for the mass function $m(p)$ and $\beta(p)$ up to order $1/N$, they show that there is a non-uniformity in the $1/N$-expansion of $\beta(p)$ for $p = 0$, so that the approximation $\beta(p) \equiv 1$ is not allowed. Pennington et al. find, in contrast to Appelquist et al., dynamical fermion mass generation for *any* value of λ.

The argument put forward by Appelquist is that, for a consistent treatment of orders of $1/N$, one has to take into account order-$1/N^2$ effects in the mass function, if one allows order-$1/N$ effects in the wave-function renormalization. Two groups have independently investigated these higher order effects in $m(p)$: Nash [58] and Atkinson, Johnson and Maris [57,60]. Both groups have arrived at the result that there *is* a critical value λ_c, deviating less than 20% from the original result of Appelquist. They claim that the non-uniformity in $\beta(p)$ disappears because of cancellation against order-$1/N^2$ terms in $m(p)$. As explained in Chapter 4, however, they have found this cancellation by making a dubious assumption concerning the series expansion in λ of $\beta(p)$.

In the last section of Chapter 4, we have chosen a different way to judge the situation. We have proposed an Ansatz for the vertex which is based on the Ward identity and which exactly satisfies the *differential* Ward identity. Compared to the Ansatz $\beta(p)\gamma_\nu$, two terms with new tensor structures are added. To do justice to the spirit of the large-N limit, we have only taken into account effects of order $1/N$ in both the equations for $m(p)$ and $\beta(p)$. The result of our (numerical) analysis is that there is no bifurcation point; dynamical fermion mass generation is allowed for any value of λ. Both qualitatively and quantitatively, our results are close to those of Pennington et al. for the Ansatz $\beta(p)\gamma_\nu$, which, apparently, embodies the leading behaviour of the vertex in the context of dynamical fermion mass generation.

Our view on the controversy on dynamical mass generation in QED$_3$(N)) is that one should not pin one's faith on higher orders in $1/N$. We think the proper way to achieve conclusive results is to treat the fermion-photon vertex non-perturbatively and as completely as possible, i.e., to respect the Ward identity fully, and furthermore consider only the model for $N \to \infty$. Our investigation has been in this spirit, and, though we did not succeed carrying out the analysis with an Ansatz for the vertex that *exactly* satisfies the Ward identity, our results indicate that no bifurcation point is to be expected in such an "exact" treatment. In this opinion we are strengthened by the results of Pennington and Walsh [64], who, in a comparable analysis, have also found dynamical fermion mass generation for *any* value of the coupling.

In Chapter 5 we have performed a similar analysis as in Chapter 4 in a *supersymmetric* extension of QED$_3$(N), for the case of SQED$_3$(N) with *one* supersymmetry ($n = 1$). We have found that the logarithmically infrared divergent contribution of order $1/N$ to the wave-function renormalization $\beta(p)$, present in non-supersymmetric QED$_3$, is cancelled by a similar but opposite term due to supersymmetry, so that the $1/N$-expansion of the supersymmetric $\beta(p)$ is uniform for any $p \geq 0$. Taking the approximation $\beta(p) \equiv 1$, which is now a consistent choice in order to investigate the matter mass function $m(p)$ to order $1/N$, we have shown that the critical number of (two-component) matter flavours, below which matter mass generation occurs, is $N_c \approx 3.24$.

In supersymmetric QED$_3$ we have obtained definitive results on dynamical mass generation. Unfortunately, we have not been able to force a final breakthrough in the controversy in the non-supersymmetric model. More research on this topic is still needed.

Another suggestion for future research is to extend the analysis of QED_3 to four-dimensional models, now that we have a basic understanding of dynamical mass generation in three dimensions. A start has already been made in QED_4 [68]. One could also think of a similar analysis of the origin of quark masses in QCD. Appelquist and Nash have initiated the investigation of QCD_3 [69], but eventually one should try to extend also the research on QCD to four dimensions. After all, the world in which we live is four-dimensional.